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Solids: A Combinatorial Auction for Real Estate

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In May 2011, our collaboration with housing corporation Stadgenoot culminated in the first combinatorial auction for housing space in a newly erected multistory building in Amsterdam in the Netherlands. Our primary goal was to allocate space based on the preferences of many potential users. The resulting allocation included space for restaurants, boutiques, a dentist, and residential users. The auction we designed maximized total rent for Stadgenoot while complying with municipal and building regulations. We based our design on laboratory experiments that gave us guidance on choices regarding, for example, pricing, feedback, and activity rules. This paper describes the development and the results of this auction.

Keywords: combinatorial auction; design; bidding; allocation constraints; real estate.
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The objective of the Stadgenoot, a Dutch housing corporation, was to construct a building in Amsterdam that would use a concept called "solids" and would last for at least 200 years. A *solid* is a sustainable building without a predefined purpose; it should accommodate any (legal) functionality, and its tenants would decide how to allocate the space within the building. Amsterdam's squatting tradition—particularly the freedom and flexibility with which squatters use the space they occupy—inspired the solids concept. The concept behind a solid is that the tenants can determine the use, size, configuration, and even the rent of their spaces. More specifically, Stadgenoot divides the floor space into lots, and tenants can combine one or more adjacent lots to create what we call a *solid space*. By choosing an appropriate set of lots (e.g., on a specific floor or with a preferred orientation), a tenant specifies the resulting solid space. Stadgenoot is, like other housing associations in the Netherlands, a nonprofit private organization with strong links to local authorities. The first housing corporations were founded in the 19th century to improve the deplorable housing conditions for working-class families at that time. Given this background, Stadgenoot's position is that solids should be open to everyone, including people with limited budgets.

The challenge we address in this paper is to develop a combinatorial auction to allow Stadgenoot to allocate its solids, taking into account various constraints and accommodating the preferences of a highly diverse group of bidders. A combinatorial auction is an auction in which multiple items are sold simultaneously, and bidders can express their preferences for sets (or subsets) of the items; Cramton et al. (2005) provide a detailed discussion of combinatorial auctions, integrating the contributions of many authors on interesting theoretical and practical aspects of these auctions. A combinatorial auction is particularly suitable for solids, because bidders are typically interested in multiple lots and may value some sets of lots higher than they value the sum of the values of the individual lots. Moreover, a lot may have no value as a single item; for example, it might not be accessible directly from the hallway. These complementarity effects are bidder specific, because bidders have different needs and preferences with respect to the space they want to rent. Some bidders may want spacious apartments with sun in the evening, other bidders want small but practical working spaces near their shops on the ground level, and still others prefer inexpensive spaces for installing offices. A combinatorial auction allows bidders to express their preferences (including budget considerations) to a greater extent than they can for individual items.

In this paper, we describe the first real estate combinatorial auction of which we are aware. Traditionally, real estate is sold as follows: an owner determines a property's characteristics, sets a price for the property, announces the property's availability, and waits for potential buyers to express interest; a private negotiation determines the actual selling price. In past decades, auctions have gradually become more popular in real estate markets worldwide. Initially, real estate auctions were most commonly used to dispose of real estate in bankruptcy cases; however, recent research shows that these auctions have earned their place alongside private negotiation as a market mechanism (Eklöf and Lunander 2003). Real estate auctions, however, typically involve a sequential and individual auctioning of properties (Quan 1994). Occasionally, in a second auction, bidders can make a single bid on all the properties that were offered in the previous auctions; the seller then selects the auction with the highest payoff. Bayers (2000) describes a case in which 14 units in a San Francisco condominium complex were auctioned simultaneously, but bidders were not allowed to submit package bids on sets of units.

Every auction requires a set of rules that determines the course of the auction and the actions bidders can take. This paper covers many auction design issues that must be decided; examples include how to motivate bidders to reveal their preferences early in the auction, what prices to charge to the winning bidders, and what feedback to provide to the bidders after each round. We settled most of these issues during meetings with Stadgenoot and fine-tuned design choices in laboratory experiments in which we tested design options. We also focus on the problem of determining an allocation that maximizes total revenue, taking into account each bid and the constraints resulting from building regulations and Stadgenoot's social policy.

Stadgenoot finished the construction of the first solid, called Furore, in April 2011; the company completed two other solids one month later and has plans to build additional buildings. In this text, we focus on Furore, which offers 7,000 square meters of floor space, features a spacious roof terrace with a splendid view of the city, and won the RIBA European Award for great architecture (Royal Institute of British Architects 2012). The auction of Furore was held in May 2011; it resulted in an allocation that is in line with the functional freedom that characterizes the solids concept and to which Stadgenoot aspires. Moreover, the company was satisfied with the revenue it received, and the solid project received positive media attention; to a substantial extent, this was a result of the novel way of allocating real estate to tenants.

More About Solids

Stadgenoot delivers solid space as a shell. This means that within the building, Stadgenoot places walls between the solid spaces, and each solid space has access to a shaft with provisions for ventilation, drainage, and electricity. However, within a solid space, the tenant decides where to place partition walls and interior doors. Thus, the rented space can to be designed to suit a range of purposes, such as residential, business, culture, or any combination of these. Stadgenoot remains the owner of the shell; the tenant rents solid space and owns the interior. A tenant who leaves can sell the interior to the next tenant. Over time, solid spaces can increase or decrease in size (by being merged with another solid space or by being split) to accommodate tenants in different ways, depending on a particular tenant's needs at the time.

A solid is divided into lots that the tenants can use as building blocks to specify solid spaces. Drawing a good allotment is a nontrivial task. The solid should comprise enough lots, with varying characteristics and sizes, such that a bidder can satisfactorily combine a number of spaces. It is also important to have enough substitutability across the lots, such that bidders can adapt the composition of their spaces depending on the course of the auction. Because building regulations and technical issues heavily constrain the allotment, we were not primarily involved in the allotment decision; therefore, we had to consider it as given. The Furore

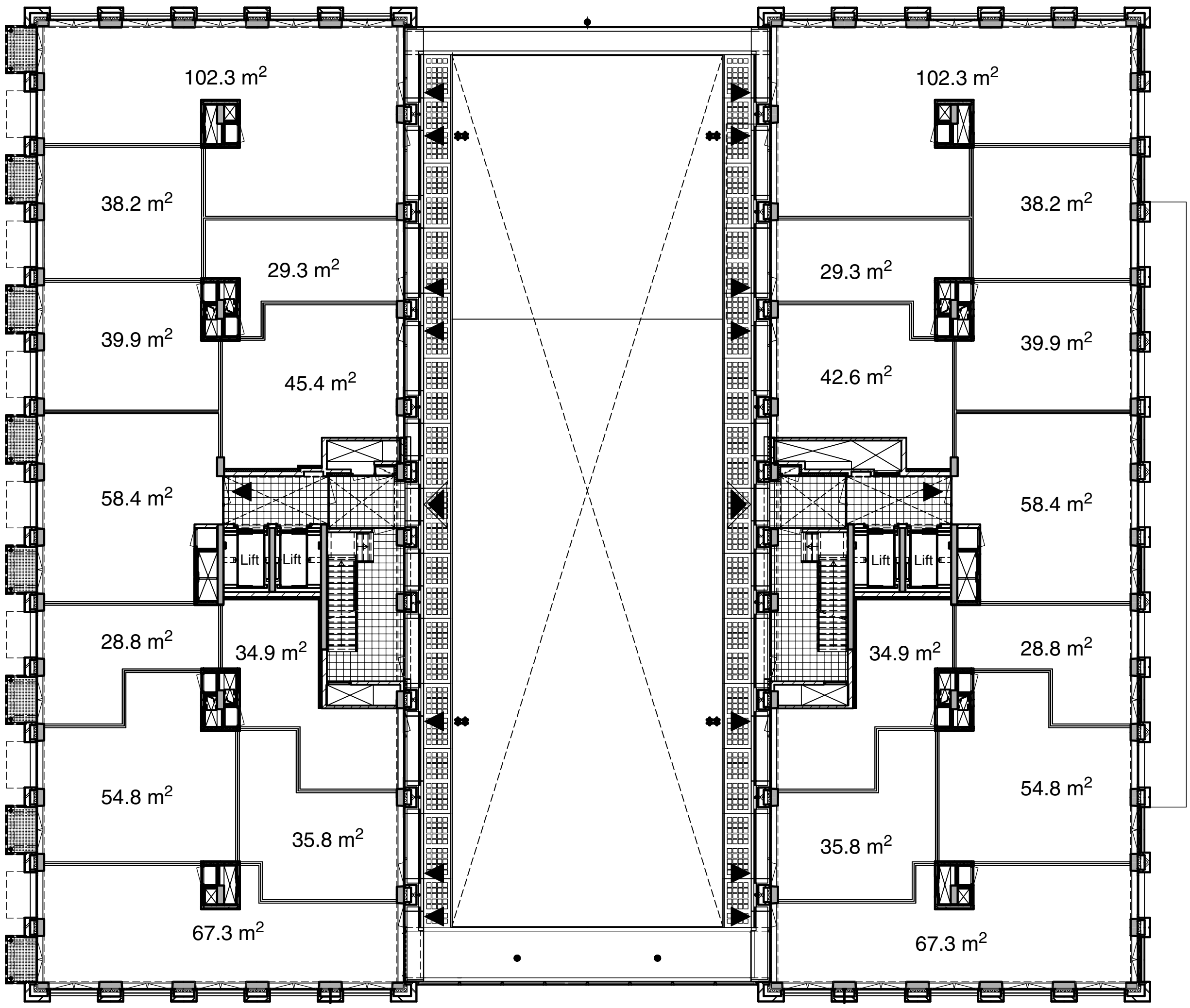

Figure 1: The allotment of the second floor of Furore illustrates how a bidder has flexibility to combine lots into a solid space that consists of the bidder's desired size and orientation.

solid was divided into 125 lots, distributed over seven floors. Figure 1 shows the allotment of the second floor, which has 22 lots, each with a given surface area.

A bid consists of a specific solid space and a price the bidder is willing to pay for it as a monthly rent (paying monthly rent is typical in the Netherlands). Although the bidders taking part in this auction may be diverse, we assume that no externalities are present in bidder's valuations (i.e., the valuation that a bidder places on a space depends only on the lots it contains). Thus, bidders do not care which bidders will be their neighbors or what their neighbors intend to do with their spaces. Stadgenoot feels that this is in line with the philosophy of the solids project. Indeed, if key concepts such as flexibility and freedom appeal to bidders, they should accept that these concepts also apply to their neighbors. Nevertheless, activities that could seriously disturb other tenants (e.g., nightclubs, heavy industry) are not allowed.

Solids suit a variety of tenants; examples include families planning to live in the building; real estate brokers looking for bargains; commercial bidders intending to open businesses (e.g., hotels, offices, shops, restaurants); and students, artists, or low-income people who want social housing. Stadgenoot categorizes bidders as residential, commercial, or social. The first group consists of people who plan to live in the solid, the second group includes people who plan to open businesses, and the third group comprises low-income people. In a combinatorial auction in its most general form, bidders can bid whatever amount they please on any subset of the items. In the solid auction, this is not the case. First, for social bidders, Dutch law imposes an upper bound of 650 euros on the monthly rent. Second, and most important, bidders cannot bid on any set of lots. A valid lot subset consists of either adjacent lots on a single floor or all lots on a floor. A bid on a set of lots on different floors is not valid. Figure 1 shows that floors above ground level consist of two wings, separated by an open space in the middle, such that lots in different wings are not adjacent. On the ground level, however, additional lots connect the wings; consequently, valid solid spaces spanning both wings are possible. Furthermore, a valid subset of lots must have at least one door to the central hallway (denoted by black triangles in Figure 1) and access to a utility shaft. The surface area of a solid space can have bounds (i.e., size limitations), depending on the type of bidder. Because of these constraints, bidders can form only 1,214 different valid spaces from the 125 lots. In the remainder of this paper, the term "solid space" refers to sets of lots that satisfy these requirements.

Apart from restrictions on the sets of lots on which bidders can bid, there are also constraints on the allocation itself. A bidder is allowed to provide multiple bids but generally can win at most one bid. Most bidders can afford only one solid space, and Stadgenoot does not want them to win bids on more lots than they want. Bidders who want multiple solid spaces are allowed to register in the auction using multiple identities and can win at most one solid space per identity. Finally, bidders can also indicate that they want to see either all or none of their bids allocated. For example, a potential hotel owner might want the top three floors or nothing at all.

Some allocation restrictions are based on Stadgenoot's historical background and philosophy. The allocation should reserve at least 15 percent of the surface to social bidders, residential bidders should be allocated at least 25 percent of the surface, and commercial bidders should be allocated at least 30 percent. A maximum of three restaurants are allowed in the solid. Other constraints originate from municipal and building regulations. For example, the stairs have an emergency rescue capacity that cannot be exceeded. The rescue capacity needed for each bid depends on the surface area of the solid space and bidder's intended function for the space. Each bid has requirements with respect to ventilation, water, gas, and electricity (again depending on the surface area and the intended function), and we can only allocate bids insofar as the capacity of the utility shaft the space uses is not exceeded. Each solid space must receive all its utilities through a single shaft; however, if a solid space contains multiple shafts, the bid winner may choose which one to use. A bidder who has no shaft preference can let the auctioneer decide, thus adding to the bid's flexibility. Each shaft goes from the ground level to the top floor; hence, its capacity may have to be split over solid spaces on different floors; the allocation should be such that this is feasible.

One of Stadgenoot's concerns was that a lack of bidders (e.g., on unpopular lots) could result in some unoccupied spaces in the solid after the auction. In such a case, the unoccupied lots should also form one or more solid spaces, such that Stadgenoot could rent them in the future. If the number of social bidders is insufficient, the number of unoccupied and suitable solid spaces should be such that when the lots are later rented to social bidders, the required 15 percent of the solid's total surface can still be achieved. Furthermore, to ensure that tenants can later occupy vacant solid spaces, we reserve sufficient emergency rescue capacity and at least one shaft with enough spare capacity to provide basic utilities.

Stadgenoot's main objective is not to make a profit but to obtain a balanced, functional mix of residential, commercial, and social tenants. Therefore, in our auction, considering the various allocation constraints is at least as important as maximizing total rent; however, we have other considerations. The auction should be accessible to everyone. Participation is a key concern in the design of any auction; the auction must be designed to attract enough bidders. Learning the auction rules can pose a challenge for a nonprofessional bidder. Hence, the auction rules should be as simple as possible, the bidding process should be user friendly, and the auction must end within a single day.

Although buying real estate is an important decision for each bidder, bidders generally prefer a shorter auction, which reduces participation costs. Stadgenoot believes that once a bidder has decided to participate, that bidder will not want a protracted process and will want to know the results as soon as possible.

Auction Design

Although combinatorial auctions have been designed and implemented successfully for school meals (Epstein et al. 2002), Mars (Hohner et al. 2003), and transportation services (Ledyard et al. 2002), and to support strategic sourcing (Sandholm et al. 2006), these designs are often problem specific and not readily practicable in our setting. More general auction mechanisms, such as the clock-proxy auction (Ausubel and Milgrom 2005) or the PAUSE procedure (Kelly and Steinberg 2000), do not answer all our above-mentioned concerns. Quan (1994) reports on the winner's choice auction for auctioning multiple real estate properties. This auction is a classic open-outcry auction, in which the winner has the right to pick any number of properties to buy at the bid's unit price. If any properties remain, a second auction with the same rules follows. Although appealing for its simplicity, this auction mechanism neglects the numerous allocation constraints we described previously.

Combinatorial auction design forces trade-offs between desirable properties (e.g., allocative efficiency, revenue maximization, transparency, computational tractability) (Pekeč and Rothkopf 2003). In the *Design Issues* section, we report on the design issues we had to address: pricing rules, duration, anchor rules, activity rules, fairness, feedback, collusion, and user friendliness. Most of the decisions were made during several meetings with Stadgenoot. We fine-tuned some decisions based on a series of laboratory experiments in which we used human bidders (see the *Laboratory Experiments* section). We discuss computational tractability in the *Winner Determination Problem* section.

Design Issues

Our discussions with Stadgenoot led to a decision to use a sealed-bid first-price auction with binding bids. In a sealed-bid auction, the bidders communicate their bids directly and solely to the auctioneer (as opposed to open-outcry auctions, in which all bids are announced publicly). We opted for a sealed-bid auction because it offers privacy and encourages participation. For example, a bidder with the highest valuation, but who is uncertain about the best competitive bid, may bid too greedily and lose. Thus, bidders with a lower valuation would still have a chance to win; consequently, they have an incentive to participate (Pekeč and Rothkopf 2003). In a first-price system, bidders pay exactly the amount they bid for the solid space they win. This was not an obvious choice; one reason is that in second-price auctions (i.e., the highest bidder pays the price of the second-highest bid), a dominant strategy is for each bidder to bid the true valuation (Vickrey 1961). Nevertheless, the transparency of the first-price concept, which considers the inexperience of the bidders in the solid auction, and the computational complexity of the Vickrey-Clarke-Groves auction (i.e., the combinatorial variant of the second-price auction) led us to adopt a first-price rule. Binding bids mean that each bid remains valid until the end of the auction; withdrawing or lowering bids is not allowed. Again, we adopted this rule because of its simplicity. Furthermore, Stadgenoot maintains a reserve price of six euros per square meter. It would prefer to leave the space empty than rent it for less than the reserve price.

Ideally, an auction ends when all winning bidders are satisfied with what they have won, and all losing bidders realize that they are unwilling or unable to pay the amount needed to obtain what they want. In a first-price auction, it suffices that no bidders bid more than they are willing to pay to realize the first part; the second part, however, cannot be guaranteed if the number of rounds is fixed before the start of the auction. Nevertheless, for practical reasons, announcing the number of rounds beforehand was exactly what Stadgenoot wanted. The number of rounds needed was less clear. We assumed one hour per round (including time for computing the allocation, processing the results, reflecting on the next action by the bidders, and collecting new bids). Given that Stadgenoot wanted the auction to complete within a single day, it decided on five rounds for half-day auctions and eight rounds for full-day auctions. The simplicity of settling the auction in a single round also appealed to Stadgenoot. The number of rounds was eventually determined based on the results of laboratory experiments we conducted, as we describe next. Having a fixed duration, however, requires strong anchor and activity rules to prevent serious bids from being delayed until the final round.

Anchor rules and activity rules fix a number of bidder decisions at some point in the auction; hence, they narrow bidder options from that point on to provide them with an incentive to reveal their preferences early in the auction (see Harsha et al. 2010). Our anchor rule restricts the solid spaces on which the bidder can submit a bid as the end of the auction nears. We consider three possibilities, in order of increasing restrictiveness:

- B1: In any round, a bidder can submit a bid on any solid space.
- B2: In the first half of the auction, a bidder can submit a bid on any solid space. In the second half of the auction, a bidder can no longer bid on new solid spaces; therefore, that bidder can only raise previous bids on solid spaces.
- B3: In the first round, a bidder is free to bid on any solid space. In the following rounds, the bidder can only raise these bids and cannot introduce bids on new solid spaces.

Our activity rule restricts the bids that bidders are allowed to raise, depending on their behavior in previous rounds. In the first auction round, bidders are free to bid on any space. In later rounds, the eligibility to raise bids depends on the activity rule, which does not apply to new bids (i.e., bids on solid spaces on which the bidder did not bid in previous rounds). In our testing, we investigate two types of activity rules:

- A1: In all rounds, bidders are free to bid on any space that the anchor rule allows. They can freely choose the amount with which they increase their bids; not increasing a bid is also a valid option.
- A2: In a specific round, bidders are active on a space if in the last round in which they are not provisional winners they raise their previous bids on this space with at least a minimum bid increment of 0.5 euros per square meter. Bidders are only allowed to raise bids on spaces on which they are active. Note that the activity of a bidder always refers to the last round in which that bidder was not a provisional winner. That is, provisional winners in a specific round do not have to bid to keep their eligibility to bid in the next round. Also note that when a bidder raises a bid with less than the minimum bid increment, that bid is valid and taken into account; however, it cannot be raised again in the next rounds.

Dealing with a target group that should be favored (e.g., social bidders) is an interesting issue. One idea is to give those bidders virtual money (i.e., a subsidy) to allow them to compete with the other bidders. Another idea is to simply reserve a percentage of the lots exclusively for bidders in the target group (i.e., a set-aside). Pai and Vohra (2012) study the pros and cons of these two possibilities. We opted for a constraint in the winner determination problem, which states that the outcome should allocate at least a given percentage of the total floor space to bidders from the target group (see the *Winner Determination Problem* section). This approach provides a stricter guarantee than a subsidy may give and does not force Stadgenoot to decide beforehand which parts of the solid to allocate to the target group.

To enable the bidders to make promising bids in the next round, we must report feedback after each round. Although computationally challenging, it is possible to compute, for each solid space in which that bidder is interested, the price that will make it a winning bid, provided that all other bids remain unchanged. Stadgenoot did not want to do this because it feared that (inexperienced) bidders would perceive this information as a kind of guaranteed winning strategy. Therefore, in our laboratory experiments, we considered the following forms of limited feedback:

- F1: After each auction round, bidders learn only which of their bids are currently in the winning allocation (if any).
- F2: After each auction round, bidders learn which spaces are allocated, and for what prices. Thus, each bidder sees the winning allocations; however, information about individual bids is anonymous.

Because we can partition the solid into solid spaces in many ways, a bidder for a solid space might benefit from bids on complementary solid spaces. This is inherent to any combinatorial auction but may encourage bidders to collude (e.g., “if you don’t bid on my solid space, I won’t bid on yours”), which can impact the total revenue generated. Although this was not Stadgenoot’s greatest concern, we provide the following measures to limit collusion: (1) reserve prices, which reduces the maximum gain of collusive bidding; (2) use sealed bids, which makes violations of the collusive agreement undetectable until it is too late for a bidder to react, at least in the final round; and (3) use binding bids with guaranteed bidder solvency.

Finally, we must remember that the vast majority of the bidders are not familiar with auctions. Therefore, user friendliness is more important than it is in an auction in which professional bidders take part. An information technology company developed an intuitive user interface to allow the bidders to express their preferences. This system shows them the allotment of the solid, including the positioning of the shafts, doors, and windows, and it allows them to select lots to form a solid space. If the selected lots do not form a valid solid space, the system immediately provides feedback. A user can also provide the system with a set of desired characteristics (e.g., surface area, orientation), and the system will generate a list of solid spaces that satisfy these characteristics. This can happen weeks before the start of the auction; interested bidders also have the opportunity to visit the solid in advance to see the space(s) on which they intend to bid.

Laboratory Experiments

We ran laboratory experiments at the Center for Research in Experimental Economics and political Decision making (CREED) laboratory at the University of Amsterdam to gain insights into the effects of various auction design factors: the number of rounds, activity rules, anchor rules, and between-round feedback. We wanted to determine how particular choices for these settings would influence the behavior of real, inexperienced bidders, and how much revenue the resulting auctions would generate. At the start of our testing, all subjects received starting capital of seven euros, which they could increase with additional payoffs (eight euros on average) depending on their performance in the auctions.

In all auctions in our experiment, 20 lots were offered for sale, corresponding to two identical floors of a building. The lots were sold in various combinatorial auction designs, each with a fixed number of rounds. After each round, we determined a winning allocation according to the formulation in the appendix; the only allocation constraint we considered was that a bidder can win at most one space. Note that the other constraints are not very meaningful in a setting with a limited number of lots and bidders; hence, they would only make the various auction design factors, which we will specify below, more confusing to the subjects.

In each auction, 8–10 subjects competed—a number that corresponded to the minimum level of competition (i.e., bidders per lot) we were expecting for the real auction. We did not rematch between auctions so that each subject competed with the same group of subjects in each of three auctions. We assigned each subject one of four roles—big business, small business, residential couple, or residential single; each had a specific target surface. Note that we ensured that demand always exceeded supply. A subject made a bid by expressing a price for a set of adjacent lots; this price had to be at least six euros per square meter. In each auction, each bidder could have bids on at most eight spaces at any time and could never revoke or lower any bid. Furthermore, bids had to satisfy the activity rule and the anchor rule.

In addition to the target, each subject received a value equal to price per square meter $\times$ target surface. The values were private information for each subject (i.e., before the auction, all subjects learned their own value; however, the only information each had about the value of the other subjects was that all prices per square meter were drawn independently from the uniform distribution on the interval $[6, 12]$). If the space in the bid consisted of a number of (highly) interesting lots, its value increased with a bonus of (40) 20 euros per lot; the bonus per lot was the same for each subject. This ensured that the subjects' preferences were not uniform for the building, and simulated the belief that some lots are more popular than others, which is likely to hold true in practice. When a subject won the target surface (or more) in the final round, the payoff equalled $3.5 \times (\text{value} - \text{bid})/(\text{target surface})$. At the end of the session, we paid the subjects their starting capital and the sum of the payoffs collected in the auctions in which they participated. After each auction, we drew new values and assigned new roles to the subjects. To ensure that the auctions were comparable, we kept the draws of the values constant across treatments.

We ran nine sessions in which we varied (1) the number of auction rounds, (2) the anchor rule, (3) the activity rule, and (4) the feedback mechanism, according to the options described in the *Design Issues* section. In each session, the subjects interacted in three successive auctions. Sessions had auctions with one, five,

Session	No. of rounds	Activity rule	Anchor rule	Feedback	No. of subjects
1	1–5–8	A1–A2	B1	F1	19 (10 + 9)
2	1–5–8	A1–A2	B2	F2	18 (9 + 9)
3	1–5–8	A2–A1	B3	F1	18 (9 + 9)
4	5–8–1	A2–A2	B1	F2	16 (8 + 8)
5	5–8–1	A2–A2	B2	F1	18 (9 + 9)
6	1–8–1	A2	B3	F2	18 (9 + 9)
7	8–1–5	A2–A1	B1	F1	16 (8 + 8)
8	8–1–5	A2–A1	B2	F2	16 (8 + 8)
9	8–1–5	A1–A2	B3	F1–F2	17 (9 + 8)

Table 1: This table provides a summary of the experimental design.

Variable	Estimate
One round	−0.1781 (0.0557)**
Eight rounds	−0.0598 (0.0447)
Activity rule A1	−0.0657 (0.0460)
Anchor rule B1	−0.1182 (0.0595)*
Anchor rule B3	−0.0785 (0.0530)
Feedback rule F1	0.0132 (0.0487)
Number of bidders	0.0137 (0.0273)
Constant	0.8401 (0.2382)
R^2 within	0.27
R^2 between	0.17
R^2 overall	0.23

Table 2: This table displays the results of a random-effects model on the auction revenue as a fraction of maximum surplus.

and eight rounds. We varied the order of the number of rounds from one session to the next. In all rounds, subjects had a fixed amount of time to submit their bids. The first round of each auction took 10 minutes; each successive round took three minutes. Note that in settings with only one auction round, the two activity rules (A1 and A2) coincide, as do the three anchor rules (B1, B2, and B3). Similarly, feedback is irrelevant in the case of single-round auctions. A full $2 \times 2 \times 3 \times 2 + 1$ factorial design was not feasible from a practical point of view. Therefore, we used a design in which we varied the four treatment factors between subjects and within subjects; Table 1 summarizes our experimental design. The second column presents the number of rounds in the three auctions of each session. The second, third, and fourth columns include the relevant activity rules, anchor rules, and feedback rules, respectively. If multiple rules are included, the first (second) refers to the first (second) multiple-round auction. The final column shows the number of subjects that participated in the session; the distribution of subjects in the two auctions is in parentheses.

We evaluated the auctions' performance on the basis of revenue as the fraction of the maximum total value subjects could obtain (i.e., the value in an optimal feasible allocation of lots over subjects). To disentangle the effect of the four auction design factors on performance, we ran a random-effects panel regression in which dummies for the factors and the number of subjects acted as explanatory variables. Table 2 presents the regressions results of a random-effects model on auction revenue as a fraction of maximum surplus. The regression analysis takes bidding group-specific effects into account, where a bidding group is the set of 8, 9, or 10 subjects that compete against each other in three consecutive auctions. The first column indicates the independent variables and the goodness-of-fit measures. The first five variables are dummies. Five rounds, activity rule A2, anchor rule B2, and feedback rule F2 are used as benchmarks. The second column contains estimates with (robust) standard errors in parentheses, and double asterisks (a single asterisk) denote significance at the one percent (five percent) level.

The regression results suggest that if we restrict our attention to the design features tested in the laboratory, the auction should have five rounds, activity rule A2, anchor rule B2, feedback rule F1, and as many bidders as possible. In the case of eight bidders, the model estimates that this auction's revenue equals about 96 percent of the maximum bidder surplus. Note that according to this regression, the number of rounds and the anchor rule have a significant impact on the auction's performance. If the auction takes only one round instead of five, the regression results indicate a revenue loss of about 18 percent. If we use anchor rule B1 instead of B3, the predicted loss in revenue is about 12 percent. Both effects are statistically and economically significant. The results with respect to feedback seem to contradict findings by Adomavicius et al. (2012), who claim that (for a continuous auction) without price feedback, bidders are unable to formulate effective bids, resulting in dead bids (i.e., bids that have no chance of winning) of 30–40 percent. However, the effects of the feedback rule (and of the activity rule) in our testing are not very pronounced, either statistically or economically.

We should be cautious in interpreting our results. The number of (independent) observations is not particularly high. Moreover, real bidders might behave differently than the subjects in the laboratory; because they were mainly students from the University of Amsterdam, they might not represent the most typical bidders in an actual solid auction. Still, we believe that we can safely advise against running a single-round auction and against using anchor rule B1. Note that anchor rule B1 allows a bidder to stay unnoticed until the auctions' final round; therefore, this rule essentially results in an auction in which all the action takes place in its final round. It appears that bidders tend to hold back if they have to make their final bids without (much) information about other bidders' preferences. Eventually, Stadgenoot opted for the auction design that is optimal according to the aforementioned regression results; however, it used feedback rule F2 instead of feedback rule F1.

Winner Determination Problem

Clearly, the auction design must be computationally feasible. In particular, the problem of deciding which bidders should win which items to maximize the auctioneer's revenue (i.e., the winner determination problem) must be tractable. Although fast exact algorithms exist for this problem (e.g., CABOB; Sandholm et al. 2005), they are unable to cope with various allocation constraints that we need in our setting. Next, we describe a number of theoretical results and report on our computational experiments.

Theoretical Results

In general, the winner determination problem is NP-hard (van Hoesel and Müller 2001) and does not allow good approximation results (Sandholm 2002); however, our winner determination problem is based on a restricted topology. In the solids auction, the lots in each wing and on each floor are arranged in two rows (see Figure 1). Notice that the lots need not be the same size or be aligned over both rows. Moreover, bidders can only bid on sets of adjacent lots. Thus, the resulting problem is as follows: given lots arranged in two rows, find an allocation that maximizes the total rent, provided that each bidder can win at most one bid and each lot can be allocated at most once. We refer to this problem as the *solid winner determination problem*, in which each bidder can bid up to b bids (SWDP-b). SWDP-b relates to a problem known in the literature as the job interval-selection problem (JISP1): given n pairs of intervals on the real line, select as many intervals as possible, such that no two selected intervals intersect and at most one interval is selected from each pair. Because JISP1 is MAX SNP-hard (Spieksma 1999), it immediately follows that SWDP-b for $b \geqslant 2$ is NP-hard, even when all lots are arranged on a single row.

An interesting case arises when $b = 1$, the SWDP with a single bid per bidder (SWDP-1). Notice that this is equivalent to a setting in which bidders have multiple bids but are allowed to win more than one bid. It is well known that if all lots are arranged in a single row, this problem is polynomially solvable. Rothkopf et al. (1998) found that the winner determination problem is solvable in polynomial time if a linear order exists among the items and bidders can only bid on subsets of consecutive items, even when we consider the first item in ordering the successor of the last (i.e., a circular order). Orlin (2011) showed that SWDP-1 remains easy if the lots are arranged in two rows.

Computational Experiments

We developed a mathematical formulation for the winner determination problem in the solid auction (see the appendix), and we implemented and solved it using IBM ILOG CPLEX, version 12.3. To evaluate our algorithm's performance, we performed a number of computational experiments on randomly generated instances based on Furore. Stadgenoot performed several studies to gain information about interested tenants; for example, it wanted to know the solid spaces in which the potential tenants were interested and their special needs (e.g., ventilation, electricity, gas). Stadgenoot was unsure about the number of participants; however, it considered more than 1,000 bidders to be unlikely. To approximate the amounts the bidders would be willing to bid, Stadgenoot looked at prices for similar nearby apartment spaces. We used this information to generate realistic instances with 50, 150, 500, 1,000, and 2,000 bidders, where each bidder expressed 5, 10, 20, and 40 bids. The main goal of these experiments was to evaluate whether the computation times were reasonable. Therefore, we considered only a single round. We solved all instances on a Windows XP-based system with two Intel Core 2.8 GHz processors.

	Bids per bidder			
Bidders	5	10	20	40
100	0.5 (0.9)	0.7 (1.1)	1.2 (2.0)	1.2 (2.3)
500	1.5 (2.4)	2.5 (4.8)	5.7 (11.8)	20.2 (55.2)
1,000	2.2 (3.8)	5.2 (11.5)	39.9 (89.1)	37.5 (118.8)
2,500	13.0 (26.0)	23.2 (55.9)	157.6 (362.2)	491.4 (836.1)

Table 3: The average and maximum (in parentheses) computation times (in seconds) illustrate that realistic instances are solvable in a reasonable amount of time.

Our experiments are available online (Goossens 2012). Table 3 summarizes the results; each line gives values that we averaged over 10 similar instances.

Table 3 shows average computation times (in seconds), followed by maximum computation times (in parentheses) for each setting. Clearly, CPLEX efficiently solves these instances, although considerable computation time differences exist among instances with the same number of bidders. These computational experiments show that we may expect our model to compute an optimal allocation within a range of 15 minutes (900 seconds), making a discrete, iterative auction tractable.

Outcome and Evaluation

On May 7, 2011, 114 bidders took part in the combinatorial auction for Furore, as we describe in the previous sections. The bidding started after the mayor of Amsterdam sounded the gong; it ended eight hours and 725 bids later with 95 percent of the Furore rented out (the remaining five percent was rented out in a more traditional fashion in the weeks following the auction). All bids are available on our website (Goossens 2012).

Table 4 summarizes the outcome of the auction. It shows that in the first round, the auctioneer received 528 bids, followed by another 197 in the next round (recall that our anchor rule prevents bids on new solid spaces after round 2). In each round, about 40 bidders won; as many as 82 bidders were winners at some point in the auction. Four bidders managed to go unnoticed until the last round; however, nine bidders were winning bidders from the first until the last round. A look at the (indexed) total rent in the winning allocation that was computed after each round shows that the auction closed with a total rent that was 67 percent higher than after the first round. The winner determination problem was solved efficiently, with all computation times below three seconds, as shown in the bottom row of the table. The computation times in the table result from solving the model in the appendix using the data (bids) available at each round in the Furore auction. After the final round, residential bidders won 26 percent of the total surface, commercial bidders won 54 percent, and social bidders were allocated 15 percent, as required.

	Round 1	Round 2	Round 3	Round 4	Round 5
Number of bids	528	725	725	725	725
Number of winners	38	39	43	41	39
Number of new winners	38	18	16	6	4
Total rent (indexed)	100	114	137	141	167
Computation time (s)	0.27	2.28	0.23	0.91	1.50

Table 4: This table presents a number of statistics for each round in the Furore auction.

Our evaluation focuses on two aspects of the auction: participation and design. The number of bidders was on the borderline for a decent competition. One consequence of this was that similar solid spaces were rented for varying prices. Several reasons influenced participation. First, the auction was held in unfavorable economic circumstances. The housing market, particularly the rental market component, has followed a downward trend in recent years (Francke 2010), and the economic and financial crisis were also negative factors. Second, the adventurous concept behind solids probably frightened some of the more conservative participants in the housing market. Finally, the auction itself inevitably also formed a threshold. The solid auction is a new concept; for a nonprofessional bidder, learning this mechanism poses a challenge.

With respect to the auction design, the substantial increase in Round 3 suggests that our anchor rule provided an incentive for people to raise their bids. Moreover, that almost 90 percent of the winning bidders had already made a provisionally winning bid in one or more of the previous rounds shows that the activity rule also worked. Most bidders found the feedback they received useful. Looking back at the auction, however, we believe the design could have been improved by warning the bidders when they expressed a dead bid (i.e., a bid that can never win in the auction because one or more other bids dominates it). In the Furore auction, this would have been computationally feasible; however, it may not be feasible in larger-scaled combinatorial auctions. Nevertheless, the solid auction worked adequately overall and resulted in a balanced functional mix, which was Stadgenoot's main objective. Moreover, Stadgenoot was pleased with the allocation and the total revenue it generated.

Managerial Implications and Conclusions

In many aspects, designing the solids auction has been a pioneer project with a unique philosophy. The concept of renting space and then creating its entire interior design, in conjunction with the concept of maximal freedom, requires open-minded tenants with some sense of initiative and adventure. Solids are also novel in the way they are allocated—that is, using a combinatorial auction. The combinatorial auction was a success, and one month after the Furore auction, two more solids were auctioned using our design.

The major managerial implications of this work are threefold. Our primary contribution is that we showed that a combinatorial auction in a real estate market is practical, even if we must respect various allocation constraints. Moreover, whereas combinatorial auctions were usually considered as too complicated for nonprofessional bidders, we showed that they can be made accessible to bidders whose auction experience does not exceed bidding on eBay. Although the auction was held during unfavorable economic circumstances, we successfully implemented our design, allocating over 13,000 m^2 at a satisfactory price, in a short period, and to many bidders.

The second implication is the understanding that preparing the bidders is immensely important. We highly recommend organizing a mock auction before the real auction to let the bidders become familiar with the bidding system interface and perhaps experience the consequences of certain bid strategies. In a combinatorial auction, a lower bid is sometimes allocated at the expense of a higher bid. When allocation constraints are present, this phenomenon occurs more frequently. Bidders should be informed in advance that this may happen, preferably with specific examples that add to their understanding of the auction. Ignoring this preparation will lead to frustration for some bidders and could possibly result in their filing legal suits.

Finally, when designing an auction, developing a set of simple and effective rules and carefully explaining them to the bidders is not sufficient. We should focus on the cognitive complexity of formulating combinatorial bids (Porter et al. 2003). In this regard, the complexity of participating in the auction and formulating sensible bids is important. We cannot overestimate the importance of the auction software. For example, it should be able to provide an overview of spaces (e.g., surface area, orientation, shape) to satisfy the bidders' needs in advance of the auction and during it. Furthermore, it should alert bidders about the consequences of their actions (e.g., with respect to the activity rule). User-friendly auction software is a key to a positive experience for bidders.

Despite the peculiarities of the solid project, we believe that this auction design can easily be generalized to other real estate settings. A key assumption is that bids can only be expressed on adjacent lots, which reduces the number of combinations and the complexity of the winner determination problem. Within this context, any setting in which space is to be allocated (e.g., building land, exposition space at a fair) could be handled with an auction design similar to the design we describe in this paper.

Appendix. Mathematical Formulation for the Winner Determination Problem

We based the integer programming formulation that we developed for our winner determination problem on a set-partitioning formulation and use the following notation. The solid, with a total surface area of A m^2, is divided into a set of lots denoted by L and has a number of utility shafts, $s \in S$. Each shaft s has a ventilation capacity of V_s m^3/h and offers G_s high-capacity gas connections and E_s high-current electricity connections. Per floor, $f \in F$ above the ground level, the stairs have a rescue capacity for $O_{f,w}$ persons for each of the two wings, $w \in W$. At most, R restaurants are allowed. We use B to denote the set of all bidders, and subsets B_r, B_c, and B_s represent the set of residential, commercial, and social bidders who should be awarded at least a fraction f^r, f^c, and f^s of the surface area of the solid, respectively. The set B_a includes those bidders who indicated that they want to win either all or none of their bids. Each bid $t \in T$ belongs to one bidder $b(t)$ and is characterized by the following parameters. We use $L(t)$ to represent the set of lots included in bid t, a_t for the surface area of this solid space, and p_t for the price that the bidder is willing to pay as monthly rent. The solid space is situated on wing $w(t)$ and floor $f(t)$, and the utility shaft that is to be used is given by $s(t)$. If the bidder does not explicitly mention which shaft to use, we duplicate the bid for each shaft contained in the bidder's solid space: the set $D(t)$ contains bid t and its duplicates. We use o_t to denote the number of persons for which rescue capacity is needed for bid t and v_t for the required ventilation capacity. We define T^R as a subset of the set of bids T containing those bids where the solid space is to serve as a restaurant. We also have subsets of T for those bids requiring a high-capacity gas (T_s^G) and (or) electricity (T_s^E) connection on shaft $s \in S$. We also add two dummy bidders, $d_s, d_r \notin B$, where the former bidder bids on each (valid) solid space small enough for social bidders and the latter bids on all other (valid) solid spaces. Both bidders always bid a price of zero (reflecting Stadgenoot's preference for renting out at the reserve price, rather than leaving space unallocated in this pioneer project), and they require some minimal ventilation and rescue capacity. We use the decision variable x_t, which is 1 if bid t is allocated and 0 otherwise, and y_b for each bidder b in B_a, which is 1 if all bidder b's bids are allocated and 0 if none is allocated:

$$\text{Maximize} \quad \sum_{t\in T} p_t x_t \tag{1}$$

$$\text{subject to} \quad \sum_{t\in T:\, l\in L(t)} x_t = 1 \quad \forall l \in L, \tag{2}$$

$$\sum_{t\in T:\, b(t)=b} x_t \leqslant 1 \quad \forall b \in B\backslash B_a, \tag{3}$$

$$\sum_{t\in T:\, b(t)\in B_r} a_t x_t \geqslant f^r A, \tag{4}$$

$$\sum_{t\in T:\, b(t)\in B_c} a_t x_t \geqslant f^c A, \tag{5}$$

$$\sum_{t\in T:\, b(t)\in B_s\cup\{d_s\}} a_t x_t \geqslant f^s A, \tag{6}$$

$$\sum_{t\in T^R} x_t \leqslant R, \tag{7}$$

$$\sum_{t\in T:\, w(t)=w,\, f(t)=f} o_t x_t \leqslant O_{f,w} \quad \forall w \in W, \forall f \in F, \tag{8}$$

$$\sum_{t\in T:\, s(t)=s} v_t x_t \leqslant V_s \quad \forall s \in S, \tag{9}$$

$$\sum_{t\in T_s^G} x_t \leqslant G_s \quad \forall s \in S, \tag{10}$$

$$\sum_{t\in T_s^E} x_t \leqslant E_s \quad \forall s \in S, \tag{11}$$

$$\sum_{t\in D(t)} x_t = y_b \quad \forall b \in B_a, \forall t \in T\colon b(t)=b, \tag{12}$$

$$y_b \in \{0,1\} \quad \forall b \in B_a, \tag{13}$$

$$x_t \in \{0,1\} \quad \forall t \in T. \tag{14}$$

The objective function (1) states that we should maximize the total rent. The first set of constraints (2) enforces the allocation of each lot. This is necessary to ensure that unoccupied lots (i.e., lots allocated to a dummy bidder) form a valid solid space. The second set of constraints (3) ensures that each bidder wins at most one solid space (except for those bidders who want to win all their bids or none at all, as well as the dummy bidders). Constraints (4)–(6) ensure that each type of bidder acquires at least a given percentage of the total surface area; the next constraint prevents more than R restaurants from winning a place in the solid. Constraints (8) guarantee that the rescue requirement is available for the stairs in each wing, and constraints (9) enforce that the ventilation capacity is respected for each shaft. Constraints (10) and (11) enforce that per shaft, no more than the available high-capacity gas and electricity connections are used. That some bidders want all their bids allocated, or none at all, is settled with constraints (12) and (13). The final set of constraints ensures that bids are fully accepted or not at all. Note that this formulation without constraints (4)–(13), and with "=" relaxed to "$\leqslant$" in constraints (2), corresponds to the SWDP-b problem.

Because we have dummy bids on all valid solid spaces, allocating the entire building to dummy bidders always satisfies constraints (2). Similarly, by including the social dummy bidder d_s in constraint (6), this constraint will not cause infeasibility. However, because of constraints (4) and (5), a feasible solution may not exist. If this happens in the first round, we recompute the round with f^r lowered to 15 percent and, if necessary, again with both f^r and f^c lowered to zero percent. In the latter case, the model will produce a feasible solution (e.g., leave the whole building empty), and the auctioneer will announce to the bidders that the current setting does not satisfy the conditions for a valid auction result. Bidders can use the second round to bid on other solid spaces, such that after the second round, constraints (4) and (5) are satisfied. Otherwise, the model returns infeasibility, and the auctioneer cancels the auction.

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Verification Letter

Coosje Brijker, Project Manager Solids, Stadgenoot, Sarphatistraat 370, 1018 GW Amsterdam, the Netherlands, writes:

"In this letter I confirm that there has been an intense collaboration between our organization Stadgenoot, and Dries Goossens and Frits Spieksma from K.U. Leuven. The goal of this collaboration was to design and implement an auction for real estate in Amsterdam, real estate that was developed by and built for Stadgenoot. The cooperation started in 2008, when Dries Goossens and Frits Spieksma wrote a feasibility study for us indicating the potential of a combinatorial auction for our situation. It culminated when on May 7 and June 25, 2011 two successful auctions were held, allocating (in total) over 13,000 m^2 to many bidders. As far as we are concerned, the cooperation was quite fruitful. Looking back, the influence the K.U. Leuven team had on the design of the auction (based on experiments), as well as the optimization methods they employed, were instrumental in the prosperity of the auctions. The benefits include:

- Achieving a highly satisfactory revenue.
- Using a transparent and fair allocation mechanism.
- Guaranteeing an outcome in accordance to our policy and building regulations.
- Being able to rent out 13,000 m^2 in a very short period.
- Obtaining experience in a new way of allocating space.

Furthermore, we appreciate the fact that the auction design and software can also be used for future Solid projects."

Dries R. Goossens is an assistant professor in the Faculty of Economics and Business Administration at Ghent University. He obtained a PhD degree in applied economics, with research in the field of combinatorial auctions. Apart from this, his main research interests are in combinatorial optimization problems as sport scheduling, personnel scheduling, procurement, transportation, and assignment problems. In his work, Goossens tries to combine theoretical contributions and practical applications.

Sander Onderstal is associate professor of economics at the University of Amsterdam. His research interests include auctions, industrial organization, and experimental economics. He has published widely in international journals such as the *Journal of Political Economy*, *Journal of Public Economics*, *International Economic Review*, *European Economic Review*, and *Economic Theory*. Onderstal has advised several ministries, public organizations, and private parties on auctions for real estate, gasoline stations, and radio frequencies for mobile telecommunications.

Jan Pijnacker is an information technology (IT) engineer currently working at the housing corporation Stadgenoot in Amsterdam. For the solids project, he was the project manager of the online auction. His main (work) passion is translating business needs into IT language, and vice versa. Pijnacker has a professional master's degree in management and information technology.

Frits C. R. Spieksma is a full professor in the Faculty of Business and Economics at Katholieke Universiteit Leuven, Belgium. His main research interests lay in operations research, especially combinatorial optimization problems and applications thereof, and range from (sport) scheduling and combinatorial auctions to assignment and transportation problems. Another recent field of interest is (computational) revealed preference, a subject in microeconomics.