4OR-Q J Oper Res (2010) 8:365–374
DOI 10.1007/s10288-010-0123-y

RESEARCH PAPER

Round robin tournaments and three index assignments

Dirk Briskorn · Andreas Drexl ·
Frits C. R. Spieksma

Received: 15 October 2009 / Revised: 9 March 2010 / Published online: 26 March 2010

Abstract Scheduling a sports league can be seen as a difficult combinatorial optimization problem. We study some variants of round robin tournaments and analyze the relationship with the planar three-index assignment problem. The complexity of scheduling a minimum cost round robin tournament is established by a reduction from the planar three-index assignment problem. Furthermore, we introduce integer programming models. We pick up a popular idea and decompose the overall problem in order to obtain two subproblems which can be solved sequentially. We show that the latter subproblem can be casted as a planar three-index assignment problem. This makes existing solution techniques for the planar three-index assignment problem amenable to sports league scheduling.

Keywords Combinatorial optimization · Computational complexity · Sports league scheduling · Round robin tournaments · Planar three index assignments

MSC classification (2000) 90C27 · 68Q25

D. Briskorn (✉)
Seminar für ABWL, Supply Chain Management und Produktion, Universität zu Köln,
Albertus-Magnus-Platz, 50923 Köln, Germany
e-mail: briskorn@wiso.uni-koeln.de

A. Drexl
Lehrstuhl für Produktion und Logistik, Christian-Albrechts-Universität Kiel, 24098 Kiel, Germany

F. C. R. Spieksma
Department of Operations Research and Business Statistics, Katholieke Universiteit Leuven,
Naamsestraat 69, 3000 Leuven, Belgium

1 Introduction

Sport league scheduling covers a huge variety of different problems arising in practice. In literature, both temporally-relaxed and temporally-constrained problems are studied. The former contains problems where the number of rounds is larger than the minimum number of rounds needed to schedule all matches, while in the latter case exactly the minimum number of rounds required to ensure a feasible schedule is given.

The focus of this paper is on round robin tournaments (RRT), where scheduling is temporally constrained. We consider a set T of n teams. If n is odd we can add a dummy team and, hence, we can assume, without loss of generality, that n is even. In an ℓRRT each team plays ℓ times against each other, either at home or away. Each team has to play at least $\lfloor \ell/2 \rfloor$ times at home against each other team. Obviously, this implies that no team can play more than $\lceil \ell/2 \rceil$ times at home and the resulting schedule is somehow balanced with respect to the venues of the matches. Furthermore, a team $i \in T$ has to play exactly once in each round and, hence, we have a set R of $\ell(n-1)$ rounds altogether.

We consider the case $\ell = 1$, that is, single round robin tournaments (SRRT), an instance of which is provided in Table 1, and the case $\ell = 2$, that is double round robin tournaments (DRRT). In a DRRT each team has to play once at home and once away against each other team.

A special case of a DRRT is the mirrored DRRT. Here, a match between teams $i, j \in T$ takes place in round p, $p \leq n-1$, at i's home if and only if a match between teams i and j takes place in round $p + n - 1$ at j's home, that is, the tournament is divided into 2 halves, each being an SRRT. Thus, for scheduling purposes, a mirrored DRRT is equivalent to an SRRT.

A variety of approaches for scheduling RRTs has been published, most of which are based on graph models; see, e.g., early work of de Werra (1980, 1982) and recent surveys by Drexl and Knust (2007) and Rasmussen and Trick (2008). A schedule can be represented as an *edge-coloring* of the complete graph consisting of n vertices with $n-1$ colors, where an edge-coloring of a graph is a coloring of the edges such that adjacent edges have different colors. Each vertex corresponds to a team and each edge represents a match between its two incident teams. Consequently, edges having the same color represent matches taking place in the same round. If we orient each edge, we obtain a feasible schedule containing the information about the round and the venue of each match.

In practice, simply finding a feasible schedule may already be difficult. Hence, finding a good schedule, in particular when faced with additional constraints, is a challenging task. We refer to Bartsch et al. (2006), Goossens and Spieksma (2009), and Durán et al. (2007) for recent examples of different types of constraints coming from

Table 1 An SRRT for $n = 6$

Round	1	2	3	4	5
Match 1	1-2	5-6	3-4	4-5	5-1
Match 2	5-3	1-4	2-5	3-1	4-2
Match 3	4-6	2-3	1-6	2-6	3-6

scheduling soccer leagues. The travelling tournament problem (see, e.g., Easton et al. (2001)) is the problem to find the RRT inducing least travel cost. Urrutia and Ribeiro (2006) mention the complexity of this problem as open. Recently, Bhattacharyya (2010) dealt with a slight generalization of the traveling tournament problem and proved it to be NP-hard. An approximation algorithm is proposed in Yamaguchi et al. (2009).

Many different techniques have been employed to tackle the scheduling of sports leagues. Nemhauser and Trick (1998), for example, use integer programming (see also Briskorn and Drexl (2009)), while Henz (1999) intercedes constraint programming. Moreover, Easton et al. (2003) and Trick (2003) propose to combine constraint and integer programming. Furthermore, much effort has been spent on neighborhood search, see, e.g., Anagnostopoulos et al. (2006), Hamiez and Hao (2001), and Henz (2004).

The outline of this paper is as follows: In Sect. 2 we define the problems considered in this paper and we present integer programming formulations. In Sect. 3 we deal with the complexity of SRRT by exhibiting a reduction from the planar three-index assignment problem. Section 4 further examines the relationship between our scheduling problem and the planar three-index assignment problem. We propose a scheduling approach based on a well-known decomposition scheme comprising two subproblems. We outline how the second subproblem can be represented as planar three-index assignment problem. The last section gives a summary and an outlook to future research.

2 Problem description

In this section we introduce the problem to find a minimum cost SRRT and a minimum cost DRRT, respectively, and the planar three-index assignment problem.

2.1 Tournament problems

Given a set T of teams and a set R of rounds with $|R| = |T| - 1$, each triple $(i, j, r) \in T \times T \times R, i \neq j$, represents a match of team i against team j at i's home in round r. An SRRT, then, corresponds to a set of $|T| (|T| - 1) /2$ triples such that (1) for each pair $(i, j) \in T \times T, i \neq j$, exactly one triple of the form (i, j, r) or (j, i, r) with $r \in R$ is chosen (implying that each pair of teams has to meet exactly once) and such that (2) for each pair $(i, r) \in T \times R$ exactly one triple of the form (i, j, r) or (j, i, r) with $j \in T \backslash \{i\}$ is chosen (implying that each team plays exactly once in each round). With each triple (i, j, r) we associate cost $c_{i,j,r}$. The minimum cost SRRT problem (MinCostSRRT) is to find an SRRT having the minimum sum of the chosen triples' cost.

An integer programming formulation of MinCostSRRT due to Trick (2003) is as follows. In this formulation $x_{i,j,r}$ equals 1 if and only if team i plays at home against team j in round r.

SRRT-IP

$$\min \sum_{i\in T}\sum_{j\in T\setminus\{i\}}\sum_{r\in R} c_{i,j,r}x_{i,j,r} \tag{1}$$

$$\text{s.t.} \sum_{r\in R}\left(x_{i,j,r}+x_{j,i,r}\right)=1 \quad \forall\, i,j\in T, i<j \tag{2}$$

$$\sum_{j\in T\setminus\{i\}}\left(x_{i,j,r}+x_{j,i,r}\right)=1 \quad \forall\, i\in T, r\in R \tag{3}$$

$$x_{i,j,r}\in\{0,1\} \quad \forall\, i,j\in T, i\neq j, r\in R \tag{4}$$

This formulation uses $n(n-1)^2$ binary variables and $3n(n-1)/2$ constraints. Objective function (1) represents the goal to minimize total cost. Constraints (2) ensure that each team plays each other team exactly once. Constraints (3) assure that each team plays exactly once in each round. It is not difficult to generalize SRRT-IP to a formulation for a DRRT. Indeed, by appropriately redefining R ($|R|=2(|T|-1)$), and by replacing constraints (2) with:

$$\sum_{r\in R} x_{i,j,r}=1 \quad \forall\, i,j\in T, i\neq j, \tag{5}$$

the corresponding formulation arises. Notice that the resulting formulation for a DRRT is only useful for tournaments that do not consist of two mirrored SRRT's.

To write down an integer programming formulation for an ℓ-RRT with arbitrary $\ell>2$, we replace constraints (2) by the following two sets of constraints:

$$\sum_{r\in R} x_{i,j,r}\geq \left\lfloor \frac{\ell}{2}\right\rfloor \quad \forall\, i,j\in T, i\neq j, \tag{6}$$

$$\sum_{r\in R}\left(x_{i,j,r}+x_{j,i,r}\right)=\ell \quad \forall\, i,j\in T, i<j. \tag{7}$$

The resulting formulation models the problem of finding a minimum-cost ℓ-RRT.

So far, when explaining the objective (1), we used the abstract term "cost". In order to emphasize the practical relevance of this model, we summarize some aspects of what the "cost" $c_{i,j,p}$ of real-world tournaments might cover, see also Briskorn and Drexl (2009).

- Teams may have preferences to play at home in a certain round. We can easily express these preferences through $c_{i,j,r}$. Let $pr_{i,r}\in\mathbb{R}$ be team i's preference to play at home ($pr_{i,r}>0$) or to play away ($pr_{i,r}<0$), respectively, in round r. A preference $pr_{i,r}$ is stronger than a preference $pr_{i',r'}$ if $|pr_{i,r}|>|pr_{i',r'}|$. Then, costs can be defined as $c_{i,j,r}=-pr_{i,r}+pr_{j,r}$, for example. Here, cost $c_{i,j,r}$ represents the difference between team j's preference and team i's preference with respect to the match team i (at home) versus team j in round r.
- Naturally, maximizing the overall attendance of an RRT is a major objective of the organizers. We can represent the economic value of the estimated attendance

by $c_{i,j,r}$. Let the estimated attendances $ea_{i,j,r}$ be given for each match of team i at home against team j in round r. For a given pair of teams i, j, estimated attendances might be time-dependent, that is, $ea_{i,j,r} \neq ea_{i,j,r'}$ might hold for $r \neq r'$. This is due to other events in the same region or the current season, for example. We can define costs as $c_{i,j,r} = -ea_{i,j,r}$ and obtain the objective to maximize total tournament's attendance. Equivalently, $c_{i,j,r}$ can be defined as the number of seats remaining empty in the stadium of team i if team i plays at home against team j in round r.

- In terms of more general problems, e.g. incorporating restrictions on the number of matches carried out at the same time in the same area, MinCostSRRT might be used as a subproblem, e.g., within a Lagrangean relaxation or a column generation framework. Then, $c_{i,j,r}$ is used to cover dual information also.
- A special case of MinCostSRRT arises when $c_{i,j,r} \in \{0, 1\}$ for each $i, j \in T$, $i \neq j$, $r \in R$. Then $c_{i,j,r} = 1$ denotes that team i cannot play team j in team i's home venue in round r, whereas $c_{i,j,r} = 0$ denotes that this is possible. In such a setting, we are interested in determining whether a feasible schedule, that is, a zero-cost schedule, exists. We refer to the case where $c_{i,j,r} \in \{0, 1\}$ as Availability Constrained SRRT, or AC-SRRT for short.

2.2 Planar three-index assignment problem

As we will see later, the so-called planar three-index assignment problem (P3AP) is intimately related to round robin tournaments. The P3AP can be described by recalling that three m-sets I, J, K are given, as well as a cost $d_{i,j,k}$ for each triple $(i, j, k) \in I \times J \times K$. The goal is to find m^2 triples such that each pair in $(I \times J) \cup (I \times K) \cup (J \times K)$ is present exactly once. We give here a formulation of P3AP as an integer program according to, e.g., Spieksma (2000) using m^3 binary variables and $3m^2$ constraints. In this formulation $x_{i,j,k}$ equals 1 if and only if triple (i, j, k) is chosen.

P3AP-IP

$$\min \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} d_{i,j,k} x_{i,j,k} \tag{8}$$

$$\text{s.t.} \sum_{k \in K} x_{i,j,k} = 1 \quad \forall\, i \in I, j \in J \tag{9}$$

$$\sum_{j \in J} x_{i,j,k} = 1 \quad \forall\, i \in I, k \in K \tag{10}$$

$$\sum_{i \in I} x_{i,j,k} = 1 \quad \forall\, j \in J, k \in K \tag{11}$$

$$x_{i,j,k} \in \{0, 1\} \quad \forall\, i \in I, j \in J, k \in K \tag{12}$$

The objective function (8) sums up the chosen triples' cost. Constraints (9), (10), and (11) force each pair in $(I \times J) \cup (I \times K) \cup (J \times K)$ to be contained exactly once in the chosen triples.

3 Complexity

Easton (2002) proved a special case of AC-SRRT to be NP-complete by a reduction from the problem to complete a partially filled latin square. In the following, we give an alternative proof by a reduction from P3AP to AC-SRRT. As we will see, our reduction gives additional insights into approximability of MinCostSRRT.

Theorem 1 *There is no constant-factor approximation algorithm for MinCostSRRT unless P=NP.*

Proof We prove the theorem by presenting a reduction from P3AP to AC-SRRT. P3AP is proved to be NP-complete in Frieze (1983).

The decision version of P3AP can be described as follows:
Input: Three m-sets I, J, K, and a set $A \subseteq I \times J \times K$.

Question: Does there exist a set $M \subseteq A$ of m^2 triples such that each pair $(i, j) \in (I \times J)$, $(i, k) \in (I \times K)$, and $(j, k) \in (J \times K)$ is contained exactly once in a triple from M?

Clearly, P3AP remains NP-complete when restricted to even m; hence we assume, without loss of generality that m is even. Given an instance of P3AP, we now build an instance of AC-SRRT as follows. There are $2m$ teams, so we have $|T| = n = 2m$ (and of course $|R| = 2m - 1$). Further, we set

$$c_{i,m+j,p} = \begin{cases} 0 & \text{for each triple } (i, j, p) \in A, \\ 1 & \text{for each triple } (i, j, p) \notin A, \end{cases}$$

and

$$c_{i,j,p} = \begin{cases} 1 & \text{for } i = 1, \dots, m,\ j = 1, \dots, m,\ i \neq j,\ p = 1, \dots, m, \\ 1 & \text{for } i = m+1, \dots, 2m,\ j = m+1, \dots, 2m,\ i \neq j,\ p = 1, \dots, m, \\ 1 & \text{for } i = m+1, \dots, 2m,\ j = 1, \dots, m,\ p = 1, \dots, m, \\ 1 & \text{for } i = 1, \dots, m,\ j = m+1, \dots, 2m,\ p = m+1, \dots, 2m-1, \\ 0 & \text{for } i = 1, \dots, m,\ j = 1, \dots, m,\ i \neq j,\ p = m+1, \dots, 2m-1, \\ 0 & \text{for } i = m+1, \dots, 2m,\ j = m+1, \dots, 2m,\ i \neq j,\ p = m+1, \dots, 2m-1, \\ 1 & \text{for } i = m+1, \dots, 2m,\ j = 1, \dots, m,\ p = m+1, \dots, 2m-1. \end{cases}$$

This completes the description of the instance of AC-SRRT.

A yes-answer to the P3AP instance corresponds to a feasible solution to AC-SRRT in the following way. First, the triples (i, j, k) that constitute the solution of P3AP give rise to the following partial solution of AC-SRRT: Team i plays team $m + j$ in round k in team i's home venue. Since in this way we use only triples from A, we have ensured that each match between a team i with $1 \leq i \leq m$, and a team j with $m + 1 \leq j \leq 2m$ is scheduled with zero cost. Second, to schedule the remaining matches, let us first deal with the matches between different teams i and j with $1 \leq i, j \leq m$. Observe that we must assign these matches to rounds $m + 1, \dots, 2m - 1$ in order to have a zero-cost solution. Assigning these matches to $m - 1$ rounds can be seen as edge-coloring a complete graph (recall that an edge-coloring of a graph is a coloring of the edges such that adjacent edges have different colors). Indeed, the graph that results when there is a vertex for each of the first m teams, and an edge for each match to be played is complete. It is well-known (see Mendelsohn and Rosa (1985)) that, in case m is even—as we assumed—$(m-1)$ colors suffice to edge-color a complete graph on m nodes. The resulting coloring gives us a feasible assignment of matches to rounds (edges with the same color correspond to matches played in the same round). In this way, each round receives $\frac{m}{2}$ matches, each with zero cost. By using the same procedure for different teams i and j with $m+1 \leq i, j \leq 2m$, we find an assignment of the corresponding matches to rounds $m+1, \ldots, 2m-1$. Total cost of these matches equals zero. Hence, we have found a feasible solution to AC-SRRT.

Finally, if a zero-cost solution to AC-SRRT instance exists, it is not difficult to show that P3AP admits a solution. Indeed, let us focus on the matches between teams i and j with $1 \leq i \leq m$ and $m+1 \leq j \leq 2m$. From the construction it is clear that the existence of a zero-cost solution implies that team j never plays at its home venue against team i since this costs 1. Hence, the assignment of matches of team i against team j to rounds p, $p = 1, \ldots, m$ (which must exist since we assumed a zero-cost solution to AC-SRRT exists), gives us the solution to P3AP.

The above implies that MinCostSRRT is NP-hard.

To prove the inapproximability of MinCostSRRT, we use a standard argument: suppose there is a constant-factor approximation algorithm A for MinCostSRRT. Then, obviously, A must provide a zero-cost solution to AC-SRRT if there is one. Hence, A gives a zero-cost solution if and only if the answer to the underlying instance of P3AP is yes. Theorem 1 follows. □

In the following section we study approaches to sports league scheduling which make use of the fact that round robin tournaments are intimately related to planar three-index assignments.

4 Scheduling approach

4.1 Problem decomposition

As outlined above the minimum cost SRRT problem is hard to solve. Therefore, solution approaches usually are based on a decomposition of the problem. A frequently used decomposition scheme is to separate the problem of finding the venue of a match and the problem of finding a match's date.

In the literature two types of decomposition schemes are predominant:

- **First-break-then-schedule:** First, for each team, the venue (home/away) in each round is fixed. Afterwards, matches are arranged such that we obtain a full tournament where matches are carried out with regard to fixed venues.
- **First-schedule-then-break:** First, pairs of teams are designated to compete in a specific round. Based on this timetable each match's venue is fixed.

In the following we focus on the second step in the first-break-then-schedule decomposition scheme. Thus, we consider the scheduling problem arising when the venues are given for each team. These fixed venues can be represented by a *home-away-pattern* (HAP) for each team. A HAP is a binary string of length $|R|$, where a 0 (1) in slot r means the corresponding team plays at home (away) in round r, $1 \leq r \leq |R|$. A set of HAPs, one for each team, is called a HAP set. A HAP set is called feasible if there is a RRT such that each team plays in each round according to the venue given by the HAP set. We denote the entry of the HAP set h corresponding to team i in slot r by $h_{i,r}$. We show that for a special case of HAP sets we can reduce the resulting scheduling problem to P3AP.

4.2 Scheduling a DRRT with a mirrored HAP set

In this section we study the problem to schedule matches in a DRRT when each team's venue in each period is predetermined. Let T and R, $|R| = 2(|T| - 1)$, be given as before. Furthermore, for each round r a partition of the set of teams into the subsets $H_r \subseteq T$ and $A_r \subseteq T$ of teams playing at home and away is predetermined. We refer to a schedule for a DRRT that is compatible with these given HAPs as a *HAP-based* DRRT. In other words, a HAP-based DRRT corresponds to a DRRT such that for each selected (i, j, r) we have $i \in H_r$ and $j \in A_r$. Again, costs $c_{i,j,r}$ are associated with each triple $(i, j, r) \in (T \times T \times R)$, $i \neq j$. The minimum cost HAP-based DRRT problem (MinCostHAPbasedDRRT) is to find a HAP-based DRRT having the minimum sum of chosen triples' cost.

We say that a HAP set h for a DRRT is *mirrored* if and only if $h_{i,r} + h_{i,r+|T|-1} = 1$ for each $i \in T$ and $1 \leq r \leq |T| - 1$.

Theorem 2 *If the underlying HAP set is mirrored, then MinCostHAPbasedDRRT can be reduced to P3AP.*

Proof For a given instance of MinCostHAPbasedDRRT we construct an instance of P3AP in the following. We assume that the HAP set induced by H_r and A_r for each round r is such that a feasible solution exists. We set $m = |T|$ and for each $i, j, r \in \{1, 2, \ldots, m\}$, we define the cost-coefficients of P3AP as follows:

$$d_{i,j,r} = \begin{cases} M & \text{if } r = m \wedge i \neq j, \\ 0 & \text{if } r = m \wedge i = j, \\ M & \text{if } r < m \wedge i \in A_r \wedge j \in A_r, \\ M & \text{if } r < m \wedge i \in H_r \wedge j \in H_r, \\ c_{i,j,r} & \text{if } r < m \wedge i \in H_r \wedge j \in A_r, \\ c_{i,j,r+|T|-1} & \text{if } r < m \wedge i \in A_r \wedge j \in H_r. \end{cases} \tag{13}$$

Now, suppose an optimal solution x to this instance of P3AP is given. The optimal value of this optimal solution value is lower than M. We now construct a solution y to the instance of MinCostHAPbasedDRRT as follows:

- for each chosen triple (i, j, r) with $r < m$ in x and $i \in H_r$ in x we choose (i, j, r) in y, and
- for each chosen triple (i, j, r) with $r < m$ in x and $i \in A_r$ in x we choose $(i, j, r + |T| - 1)$ in y.

Notice that each triple (i, j, m), $i = j$, is chosen in x for any $1 \le i \le m$. It follows that for each $1 \le i, j \le m$, $i \ne j$, a triple (i, j, r), $r < m$, must have been chosen in x and, therefore, in y, we have a triple (i, j, r), $1 \le r \le 2|T|$, for each pair of teams.

Since each $1 \le i \le |T|$ is involved in exactly one triple (i, j, r) and in exactly one triple (j, i, r) for each $r < m$ in x, we have exactly one triple (i, j, r) or (j, i, r) for each $1 \le r \le 2|T|$ in y.

It is easy to see that there is a one-to-one correspondence between solutions to the instance of P3AP having cost lower than M and solutions to the instance of MinCostHAPbasedDRRT. By construction of $d_{i,j,r}$, the solution values of corresponding solutions are equal to each other, which implies Theorem 2. □

Theorem 2 enables us to employ techniques and algorithms developed for P3AP (see e.g. Magos and Miliotis 1994; Magos 1996) to solve the second step in the first-branch-then-schedule decomposition scheme.

Obviously, the reduction presented above is suitable for mirrored HAP sets only. However, it is easy to see that a slight modification makes the reduction applicable to the more general case of HAP sets being a permutation of columns of a mirrored HAP set.

5 Summary and future work

In this paper we have examined the close relationship between RRTs and planar three-index assignments. In particular, we proved that finding a schedule to a single round robin tournament comprises an NP-hard optimization problem. Moreover, we have shown that planar three-index assignment problems play a vital role in first-break-then-schedule decomposition schemes when the venues of matches have already been fixed.

The results exhibited in this paper open up several avenues for future research in sports league scheduling, to mention a few: first, the development of tailored exact and heuristic algorithms addressing the special structure of planar three-index assignment problems arising in this context. Second, embedding such algorithms in (sequential) first-break-then-schedule decomposition schemes. Third, the formulation of simultaneous optimization models covering break requirements, too. Last but not least, the development of, e.g., Lagrangean relaxation based approaches in order to get tight lower and upper bounds for these optimization models.

Acknowledgments We thank the referees for their constructive comments.

References

Anagnostopoulos A, Michel L, van Hentenryck P, Vergados Y (2006) A simulated annealing approach to the travelling tournament problem. J Schedul 9:177–193

Bartsch T, Drexl A, Kröger S (2006) Scheduling the Professional Soccer Leagues of Austria and Germany. Comput Oper Res 33:1907–1937

Bhattacharyya R (2010) A note on complexity of traveling tournament problem. Technical report, Indian Statistical Institute

Briskorn D, Drexl A (2009) Integer programming models for round robin tournaments. Comput Oper Res 36(3):837–852
de Werra D (1980) Geography, games and graphs. Discrete Appl Math 2:327–337
de Werra D (1982) Minimizing irregularities in sports schedules using graph theory. Discrete Appl Math 4:217–226
Drexl A, Knust S (2007) Sports league scheduling: graph- and resource–based models. Omega 35:465–471
Durán G, Guajardo M, Miranda J, Sauré D, Weintraub A (2007) Scheduling the Chilean Soccer League by integer programming. Interfaces 37:539–552
Easton K (2002) Using integer programming and constraint programming to solve sports scheduling problems. PhD thesis, Georgia Institute of Technology, USA
Easton K, Nemhauser G, Trick M (2001) The travelling tournament problem: description and benchmarks. In: Walsh T (ed) Proceedings of principles and practice of constraint programming—CP 2001, pp 580–585
Easton K, Nemhauser G, Trick M (2003) Solving the travelling tournament problem: a combined integer programming and constraint programming approach. In: Burke E, de Causmaecker P (eds) Proceedings of the 4th international conference on the practice and theory of automated timetabling, lecture notes in computer science 2740. Springer, pp 100–109
Frieze A (1983) Complexity of a 3–dimensional assignment problem. Eur J Oper Res 13:161–164
Goossens D, Spieksma F (2009) Scheduling the Belgian Soccer League. Interfaces 39:109–118
Hamiez J-P, Hao J-K (2001) Solving the sports league scheduling problem with Tabu Search. In: Lecture notes in computer science 2148. Springer, pp 24–36
Henz M (1999) Constraint–based round robin tournament planning. In: de Schreye D (ed) Proceedings of the international conference on logic programming. MIT Press, Las Cruces, pp 545–557
Henz M (2004) Playing with constraint programming and large neighborhood search for travelling tournaments. In: Burke E, Trick M (eds) Proceedings of the 5th international conference on the practice and theory of automated timetabling, pp 23–32
Magos D (1996) Tabu search for the planar three–index assignment problem. J Glob Optim 8:35–48
Magos D, Miliotis P (1994) An algorithm for the planar three–index assignment problem. Eur J Oper Res 77:141–153
Mendelsohn E, Rosa A (1985) One factorizations of the complete graph—a survey. J Graph Theory 9:43–65
Nemhauser GL, Trick MA (1998) Scheduling a major college basketball conference. Oper Res 46:1–8
Rasmussen RV, Trick MA (2008) Round robin scheduling—a survey. Eur J Oper Res 188:617–636
Spieksma FCR (2000) Multi index assignment problems: complexity, approximation, applications. In: Pitsoulis L, Pardalos P (eds) Nonlinear assignment problems, algorithms and applications. Kluwer, Dordrecht, pp 1–12
Trick MA (2003) Integer and constraint programming approaches for round robin tournament scheduling. In: Burke E, de Causmaecker P (eds) Proceedings of the 4th international conference on the practice and theory of automated timetabling, lecture notes in computer science 2740. Springer, Berlin, pp 63–77
Urrutia S, Ribeiro CC (2006) Maximizing breaks and bounding solutions to the mirrored traveling tournament problem. Discrete Appl Math 154:1932–1938
Yamaguchi D, Imahori S, Miyashiro R, Matsui T (2009) An improved approximation algorithm for the traveling tournament problem. Technical report, University of Tokyo